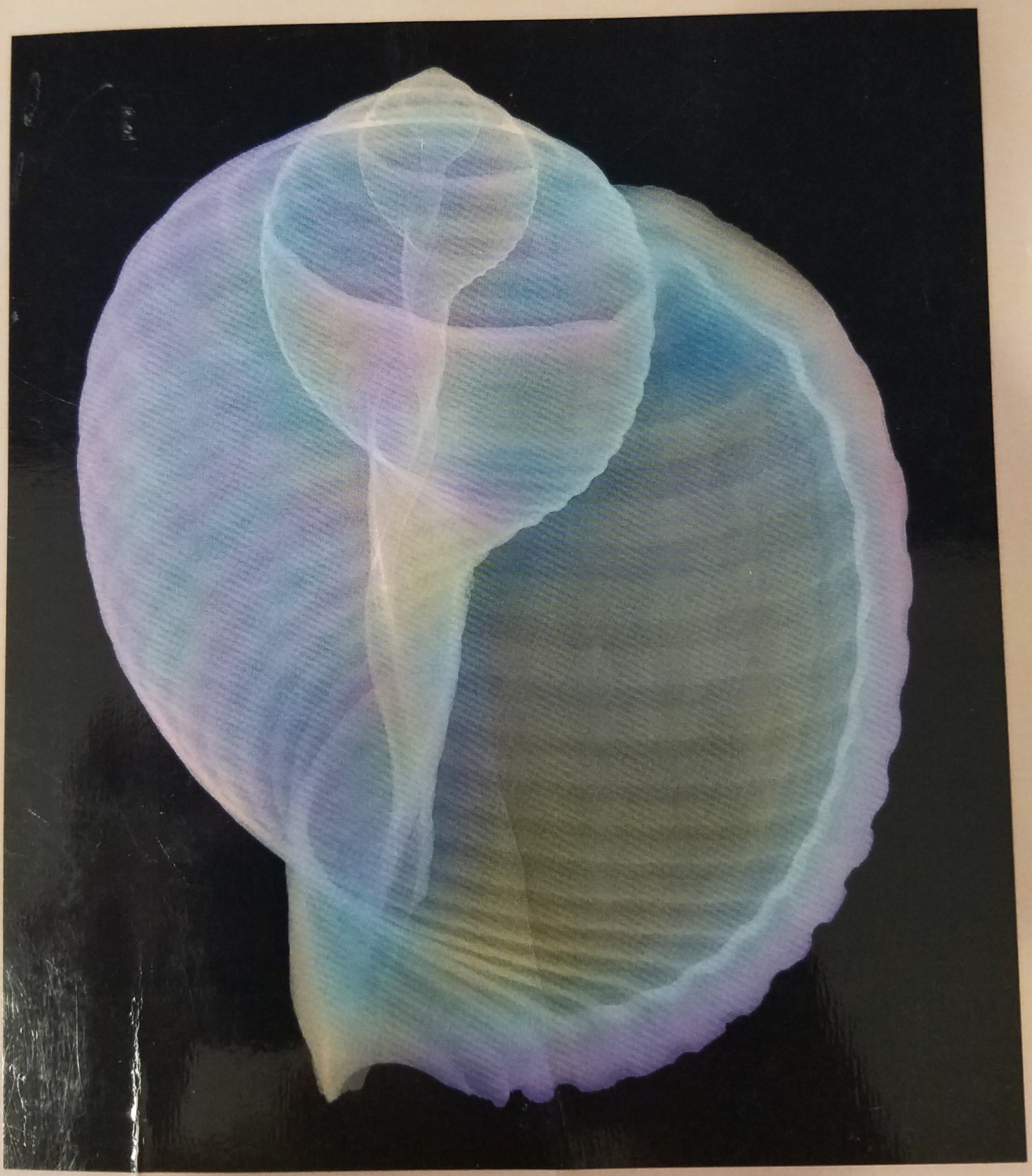
EXHIBIT 8

Introduction to Mathematical Statistics

Sixth Edition



Hogg · McKean · Craig

Introduction to Mathematical Statistics

Sixth Edition

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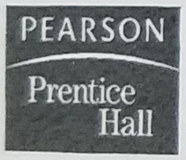
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Chapter 3

Some Special Distributions

3.1 The Binomial and Related Distributions

In Chapter 1 we introduced the *uniform distribution* and the *hypergeometric distribution*. In this chapter we discuss some other important distributions of random variables frequently used in statistics. We begin with the binomial and related distributions.

A **Bernoulli experiment** is a random experiment, the outcome of which can be classified in but one of two mutually exclusive and exhaustive ways, for instance, success or failure (e.g., female or male, life or death, nondefective or defective). A sequence of **Bernoulli trials** occurs when a Bernoulli experiment is performed several independent times so that the probability of success, say p, remains the same from trial to trial. That is, in such a sequence, we let p denote the probability of success on each trial.

Let X be a random variable associated with a Bernoulli trial by defining it as follows:

$$X(\text{success}) = 1 \quad \text{and } X(\text{failure}) = 0.$$

That is, the two outcomes, success and failure, are denoted by one and zero, respectively. The pmf of X can be written as

$$p(x) = p^x(1-p)^{1-x}, \quad x = 0, 1, \tag{3.1.1}$$

and we say that X has a *Bernoulli distribution*. The expected value of X is

$$\mu = E(X) = \sum_{x=0}^{1} xp^x(1-p)^{1-x} = (0)(1-p) + (1)(p) = p,$$

and the variance of X is

$$\begin{aligned} \sigma^2 = \text{var}(X) &= \sum_{x=0}^{1}(x-p)^2 p^x(1-p)^{1-x} \\ &= p^2(1-p) + (1-p)^2 p = p(1-p). \end{aligned}$$

It follows that the standard deviation of X is $\sigma = \sqrt{p(1-p)}$.

In a sequence of n Bernoulli trials, we shall let X_i denote the Bernoulli random variable associated with the ith trial. An observed sequence of n Bernoulli trials will then be an n-tuple of zeros and ones. In such a sequence of Bernoulli trials, we are often interested in the total number of successes and not in the order of their occurrence. If we let the random variable X equal the number of observed successes in n Bernoulli trials, the possible values of X are $0, 1, 2, \ldots, n$. If x successes occur, where $x = 0, 1, 2, \ldots, n$, then $n - x$ failures occur. The number of ways of selecting the x positions for the x successes in the n trials is

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}.$$

Since the trials are independent and the probabilities of success and failure on each trial are, respectively, p and $1 - p$, the probability of each of these ways is $p^x(1-p)^{n-x}$. Thus the pmf of X, say $p(x)$, is the sum of the probabilities of these $\binom{n}{x}$ mutually exclusive events; that is,

$$p(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, 2, \ldots, n \\ 0 & \text{elsewhere.} \end{cases}$$

Recall, if n is a positive integer, that

$$(a+b)^n = \sum_{x=0}^{n} \binom{n}{x} b^x a^{n-x}.$$

Thus it is clear that $p(x) \geq 0$ and that

$$\begin{aligned} \sum_x p(x) &= \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} \\ &= [(1-p)+p]^n = 1. \end{aligned}$$

Therefore, $p(x)$ satisfies the conditions of being a pmf of a random variable X of the discrete type. A random variable X that has a pmf of the form of $p(x)$ is said to have a **binomial distribution**, and any such $p(x)$ is called a **binomial pmf**. A binomial distribution will be denoted by the symbol $b(n, p)$. The constants n and p are called the **parameters** of the binomial distribution. Thus, if we say that X is $b(5, \frac{1}{3})$, we mean that X has the binomial pmf

$$p(x) = \begin{cases} \binom{5}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x} & x = 0, 1, \ldots, 5 \\ 0 & \text{elsewhere.} \end{cases} \tag{3.1.2}$$

The mgf of a binomial distribution is easily obtained as follows,

$$\begin{aligned} M(t) &= \sum_x e^{tx} p(x) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x (1-p)^{n-x} \\ &= \sum_{x=0}^{n} \binom{n}{x} (pe^t)^x (1-p)^{n-x} \\ &= [(1-p) + pe^t]^n \end{aligned}$$